

SENIOR HIGH MATH LEAGUE
April 24, 2001

GROUP IV Emphasis on TRIGONOMETRY

TEST A

Write all radical expressions in simplified form and unless otherwise stated give exact answers.

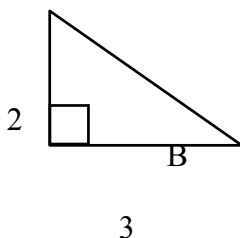
SECTION I: ONE POINT EACH

1. Give the exact value for each of the following where the angle is given in radians:

(a) $\cos\left(\frac{19\pi}{4}\right)$ (b) $\cot\left(\frac{-5\pi}{3}\right)$

2. Give the exact value for the following: $\cos\left(\arcsin\left(-\frac{3}{5}\right)\right)$

3. Use the triangle below to give the exact values of the following:



- (a) $\sec \angle B$
(b) $\cot \angle B$
4. At the points where all functions listed are defined $\sin(2\theta)$ is equivalent to which of the following expressions?

(a) $2 \cos^2 \theta - 1$ (b) $2 \sin \theta \cos \theta$ (c) $\frac{2 \sin^2 \theta - 1}{-\cos^2 \theta - \sin^2 \theta}$
(d) all of these (e) none of these

SECTION II: TWO POINTS EACH

1. For the function of θ given by $7 \cos\left(\frac{2\theta}{3} + 3\pi\right)$ find exact values for the following when they exist; otherwise, put doesn't exist. The angle is given in radians:

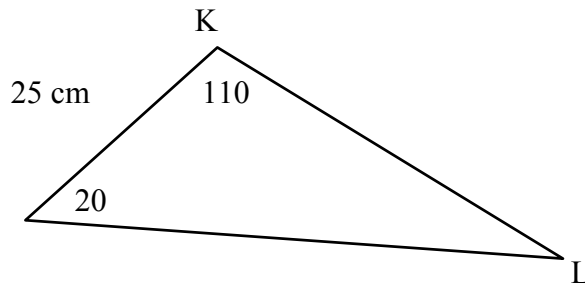
- (a) The period.
(b) The amplitude.

2. Given $\sec(2\theta) = \frac{4}{3}$ and $0 \leq \theta \leq \frac{\pi}{4}$, find the exact value of each of the following where θ is given in radians:

(a) $\cos^2(\theta)$ (b) $\sin(2\theta)$

3. Two inlet pipes can fill a water basin in 10 hours and 12 hours respectively when open individually. If an outlet pipe can empty the basin alone in 9 hours, how long would it take to fill the basin if all three pipes are open simultaneously? (Express the solution as a fraction.)

4. Find the approximate length of KL given that the angles are in degrees and the side in centimeters. Express your answer in decimal form and round your answer to the nearest tenth of a degree.



5. Find the equation of a line passing through (2, 5) and parallel to the line given by $4x - y = 7$. Express your answer in the form $y = mx + b$.

6. A student scored 60% on a 20 question exam. The instructor let him work an additional 20 problems to improve his score. How many must the student get right to raise his grade to 70%?

7. Evaluate the following expression exactly where the angles are given in degrees.

$$\sin^2(10^\circ) + \sin^2(20^\circ) + \sin^2(30^\circ) + \dots + \sin^2(60^\circ) + \sin^2(70^\circ) + \sin^2(80^\circ)$$

8. A rectangle has perimeter 14 inches and diagonal 5 inches. Find its area.

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TEST B

Write all radical expressions in simplified form and unless otherwise stated give exact answers.

SECTION I: ONE POINT EACH

1. Give the exact value for each of the following and simplify where in part a) the angles are given in radians and in part b) they are given in degrees:

(a) $2 \cos \frac{3\pi}{2} - 2 \cot \frac{\pi}{2}$ (b) $\tan(245^\circ) \cot(115^\circ)$

2. Find all integers x satisfying $4x > -x + 5 > 3x - 4$.
3. Find the exact area of a triangle whose sides are given by 5 cm, 8 cm, and 12 cm. Express your answer in simplest form.
4. Find the ordered pair (x,y) with the smallest possible y value that satisfies both $y \geq x^2$ and $y \geq x + 2$.

SECTION II: TWO POINTS EACH

1. Semicircles are constructed using the three sides of a right triangle as diameters. If the hypotenuse of the triangle measures d units, what is the total area of the three semicircles in terms of d ?
2. Two cars leave from the same place at 1:00 p.m.. One car's speed is 65 miles/hour and its heading is 65 degrees. The other car's speed is 55 miles/hour and its heading is 85 degrees. How far apart are the two cars at 3 p.m. ? Give your answer in miles, in decimal form, and to three significant digits.
3. Find the area of a regular hexagon inscribed in a circle of radius 4 meters.
4. Solve the equation $2 \sin 2\theta - 2\sqrt{3} \cos \theta = 0$ on the interval $0 \leq \theta < 2\pi$. Express your answers as exact values in radians and in increasing order of magnitude.
5. A square is inscribed in a circle of diameter 6 inches. If a point is chosen at random within the circle what is the probability that it also falls within the square ? Give an exact answer expressed in fraction form in terms of the number π .
6. A number n is added to the numerator and the denominator of the fraction $3/5$ and the result is a fraction with value $4/5$. Find the number n .

7. A pilot is traveling at a rate of 250 miles per hour and rising 25 feet per second. What is the angle his plane is making with the horizon? Your answer must be degrees and given to the nearest tenth of a degree in decimal form.
8. A password for the new computer system at CofO will require a letter followed by three digits (digits may be repeated). How many passwords are possible under this system?

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Answer Form A

SECTION I: ONE POINT EACH

1. (a) _____
(b) _____
2. _____
3. (a) _____
(b) _____
4. _____

SECTION II: TWO POINTS EACH

1. (a) _____
(b) _____
2. (a) _____
(b) _____
3. _____
4. _____
5. _____
6. _____
7. _____
8. _____

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Answer Form B

SECTION I: ONE POINT EACH

1. (a) _____

(b) _____

2. _____

3. _____

4. _____

SECTION II: TWO POINTS EACH

1. _____

2. _____

3. _____

4. _____

5. _____

6. _____

7. _____

8. _____